MTH 151 (36 points) Fall 2023, Exam 1

Name: ______________________ #(   ) October 3, 2023

No notes or other aids are allowed. Read all directions carefully and write your answers in the space provided. **To receive full credit, you must show all of your work. The final answer without steps/work is only worth 0.25 of the points of the question**

1. Find the anti-derivative

   (a) (2 points) $\int x^3(x+4)\,dx$

   $= \int x^4 + 4x^3\,dx$

   $= \frac{x^5}{5} + x^4 + C$

   (b) (2 points) $\int \frac{x^3+1}{x^4}\,dx$

   $= \int \frac{x^3}{x^4} + \frac{1}{x^4}\,dx$

   $= \int \frac{1}{x} + x^{-4}\,dx$

   $= \ln|x| + \frac{x^{-3}}{-3} + C$

2. (3 points) A particle moves along a straight line with the acceleration function

$$a(t) = 2t + 4 \ \ (m/s^2)$$

Find the velocity $v(t)$ of the particle if it is known that its initial velocity $v(0) = 3\,m/s$?

$v(t) - v(0) = \int_0^t a(\cancel{x})\,d\cancel{x} = \int_0^t 2x + 4\,dx$

$v(t) - 3 = x^2 + 4x \Big|_0^t = t^2 + 4x$

$v(t) = t^2 + 4x + 3$

3. Evaluate the following integral

(a) (4 points) $\int_{e}^{e^4} \frac{\sqrt{\ln x}}{x}\, dx$ du

let $u = \ln x \longrightarrow du = \frac{dx}{x}$

$x = e \rightarrow u = \ln e = 1$

$x = e^4 \rightarrow u = \ln e^4 = 4$

$$\int_{x=e}^{x=e^4} \frac{\sqrt{\ln x}}{x}\, dx = \int_{u=1}^{u=4} \sqrt{u}\, du = \frac{2u^{3/2}}{3}\Big|_{u=1}^{u=4}$$

$$= \frac{2}{3}\left(4^{3/2} - 1^{3/2}\right)$$

$$= \frac{2}{3}(8-1) = \frac{14}{3}$$

(b) (3 points) $\int \frac{e^{-x}}{1+e^{-x}} dx$

let $\boxed{u = 1+e^{-x}} \longrightarrow du = -e^{-x}\, dx$

$\boxed{e^{-x}\, dx = \frac{du}{-1}}$

$$\int \frac{e^{-x}}{1+e^{-x}}\, dx = \int \frac{1}{u} \frac{du}{-1}$$

$$= -\ln|u| + C$$

$$= -\ln(1+e^{-x}) + C$$

4. (4 points) Evaluate the following integral $\int \tan x\,(\sec^2 x - 1)\,dx$

$\int \tan x(\sec^2 x - 1)\,dx$

$= \int \tan x \sec^2 x - \tan x \; dx$

$= \int \tan x \sec^2 x\,dx - \int \tan x\,dx$

$u = \tan x \rightarrow du = \sec^2 x\,dx$

$= \int u\,du - \int \tan x\,dx$

$= \frac{u^2}{2} - \ln|\sec x| + C$

$= \frac{\tan^2 x}{2} - \ln|\sec x| + C.$

5. (5 points) Consider the function $f(x) = e^{\sin(t)}\cos(t)$ on the interval $[0, \pi/2]$. Find the average value, f_{ave}, of the function f on the given interval. (Give the exact answer. NO approximation!).

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2} e^{\sin t}\cos t\,dt$$

let $u = \sin t \longrightarrow du = \cos t\,dt$

$t = 0 \rightarrow u = \sin 0 = 0$

$t = \pi/2 \rightarrow u = \sin\frac{\pi}{2} = 1$

$$= \frac{1}{\pi/2}\int_0^1 e^u\,du$$

$$= \frac{2}{\pi}\, e^u \Big|_0^1$$

$$= \frac{2}{\pi}\left(e^1 - 1\right).$$

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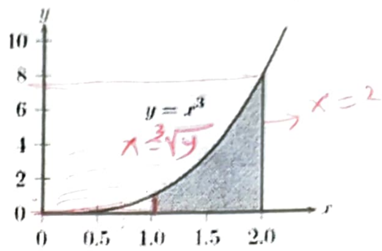
6. A region R is bounded by the curves $y = x^3$, the x-axis, $x = 1$ and $x = 2$. **Set up the integral(s)**

(a) (2 points) needed to find the area of the region R using integration with respect to x (**Don't integrate**).

$Y_{top} - Y_{bot}$

$$A(x) = \int_{1}^{2} x^3 - 0 \, dx$$

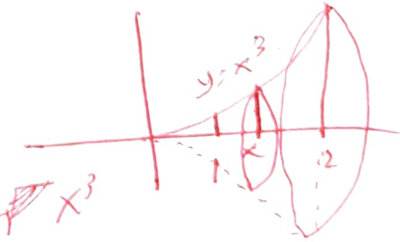


(b) (2 points) needed to find the area of the region R using integration with respect to y (**Don't integrate**).

$x_{left} - x_{right}$ $\qquad$ $x_{left} - y_{right}$

$$A(y) = \int_{0}^{8} 2 - \sqrt[3]{y} \, dy - \int_{0}^{1} 1 - \sqrt[3]{y} \, dy$$

(c) (2 points) needed to find the **volume** of the solid generated by revolving R about the x-axis.
(**Don't integrate**).



$$V(x) = \int_{1}^{2} A(x) \, dx, \quad r = x^3$$

$$= \int_{1}^{2} \pi (r)^2 dx = \int_{1}^{2} \pi (x^3)^2 dx$$

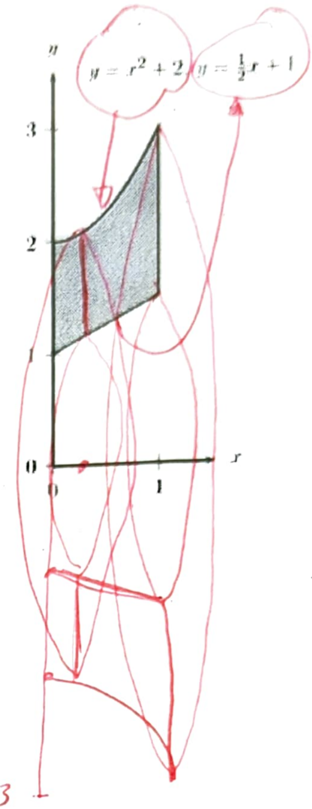
$$= \int_{1}^{2} \pi x^6 dx$$

7. A region R is bounded by the curves $y = x^2 + 2$, $y = \frac{1}{2}x + 1$, $x = 0$, and $x = 1$.

(a) (3 points) **Set up the integral** to find the **volume** of the solid generated by revolving R about the x–axis. (**Don't integrate**).

$A(x) = \pi r_{out}^2 - \pi r_{in}^2$

$$V(x) = \int_0^1 \pi(x^2+2) - \pi\left(\tfrac{1}{2}x+1\right)^2 dx$$



(b) (4 points) **Set up the integral** to find the **volume** of the solid generated by revolving R about the $y = 3$. (**Don't integrate**).

$$V(x) = \int_0^1 \pi\left(3-\left(\tfrac{1}{2}x+1\right)\right)^2 - \pi\left(3-(x^2+2)\right)^2 dx$$